## Problem 1 - Exercise 1.1

1. *What are the least and greatest number of leaf nodes in a binary tree with n nodes?*
2. *What is the relationship between the number of nodes in a full binary tree and the number of leaf nodes?*

### Solution

1. The least number of leaf nodes in a binary tree is 1. This is because when a binary tree is skewed, we only have the last node which is the leaf node.

The max number of leaf nodes can be seen in a complete binary tree with nodes n. This is given by the floor value of

Therefore, the number of leaf nodes in a binary tree with n nodes is given by 1 to , where n is positive number.

1. Let

Number of nodes = n

Number of leaf nodes = l

Height of the tree = h

Then we know that in a full binary tree:

Solving, the first equation:

We know that . Therefore,

## Problem 2

1. *Insert the following 15 randomly generated objects into a binary search tree in the order they are listed. (20 p)*

*34, 15, 65, 62, 69, 42, 40, 80, 50, 59, 23, 46, 57, 3, 29*

1. *Give two integers that could be inserted into this tree that would increase the height of this tree.*
2. *Remove the root node four times by copying up the smallest element of the right sub-tree, show the final tree*

### Solution

A close up of a watch

Description automatically generated

A close up of a necklace

Description automatically generated

A close up of a necklace

Description automatically generated

A close up of a necklace

Description automatically generated

A close up of a necklace

Description automatically generated

1. The height of the tree can be increased by adding any number from 51 to 58. This will generate a child node for 57 and increase the height by 1. The current height of the tree is 6 and then should increase by 7.

Numbers that can be added: 51, 52, 53, 54, 55, 56, 58

Note:

* Adding 2 numbers from the above set, say 51 and 52 will increase the height by 2.
* Adding numbers like 60 and 61 will increase the height by 1.

1. Deletion of root node 1st time:

A close up of a mans face

Description automatically generatedA close up of a necklace

Description automatically generated

Deletion of root node 2nd time:

A necklace with a piece of paper

Description automatically generated A necklace with a piece of paper

Description automatically generated

A necklace with a piece of paper

Description automatically generated A close up of a necklace

Description automatically generated

Deletion of root node 3rd time:

A close up of a necklace

Description automatically generated A close up of a necklace

Description automatically generated

A close up of a map

Description automatically generated

Deletion of root node 4th time:

A close up of a necklace

Description automatically generated

A close up of a clock

Description automatically generated

## Problem 3

*Insert the following n objects, in the order given, into a binary min-heap and place your answer into the following table.*

*5, 3, 9, 7, 2, 4, 6, 1, 8*

### Solution

A close up of a necklace

Description automatically generated

A close up of a mans face

Description automatically generated

A picture containing scissors, object, watch

Description automatically generated

A close up of a mans face

Description automatically generated

Required Array:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
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